THE UNIVERSITY OF TORONTO - MISSISSAUGA
Term Test 2
MAT136 H5S - Winter 2021 - All sections

Time: 2 hours
Date: Friday March 12, 2021. 4:10PM - 6:10PM (EST)
Instructors: N. Askaripour, J. Herman, J. de Jong, M. Pawliuk, S. Shen, X. Tang, Q. Wang, Z. Wang.
Aids: Course notes, Course textbook, non-programmable calculator.

Submission

- **You must submit your completed solutions on Crowdmark by 6:10pm EST Friday March 12, 2021**.
- Late submissions will not be accepted.
- You should start uploading your solutions no later than 5:30pm.
- If you require additional space, please insert extra pages.
- You do not need to print out this test; you may submit clear pictures/scans of your work on lined paper, or screenshots/PDFs of your work.

Additional Instructions

You must justify and support your solution to each question.

Permitted Resources

During the test:

- You may use any resources (course notes, textbook, videos) that have been posted to Quercus by instructors or TAs.
- You may use a non-programmable calculator.
- You may use personal notes related to official course material (from reading the textbook, participating in lectures/tutorials, posted course videos, completing WeBWork and Written Assignments).
- You may contact the instructors on Piazza using a private post.
- Do not use personal notes related to other material (e.g. notes created by studying external websites)
- Do not communicate with anyone other than the instructors.
- Do not use any online resources other than Piazza, Quercus, and Crowdmark.

Academic Integrity

Between 3:00pm-8:00pm on the day of the test you must fill out the declaration of academic integrity.

- Go to `http://declaration.utm.utoronto.ca`
- Sign-in using your U of T login.
- Enter "MAT136HS" and your full legal name.

If you do not do this you may receive a penalty of 10% on your term test.

You should not discuss this test with anyone else while the test is happening.
By submitting this test I affirm that this test represents entirely my own efforts. I confirm that:

- I have not copied any portion of this work.
- I have not allowed someone else in the course to copy this work.
- I did not discuss this test with anyone during the test.
- I understand the consequences of violating the University's academic integrity policies as outlined in the *Code of Behaviour on Academic Matters*.

Submission Checklist

This does not need to be submitted. It is for your own use.

☐ I have completed the declaration of academic integrity at `http://declaration.utm.utoronto.ca`

	Finished	Digitized	Uploaded in correct place, order, and orientation
Question 1	☐	☐	☐
Question 2	☐	☐	☐
Question 3	☐	☐	☐
Question 4	☐	☐	☐

☐ I have submitted everything in the right place, by the deadline. Time to rest.

Instructions: There are four (4) long answer questions worth five (5) points each, with multiple parts. Provide a complete solution, with justification.

Q1 - Week 6 (5 points)

(1) (2 points) Using a u-substitution, rewrite the integral $\int \sin^{1234}(\theta)\cos^{4321}(\theta)\,\mathrm{d}\theta$ as an integral of a polynomial in u (in other words, so that no trigonometric functions appear). Do not evaluate the integral.

(2) (3 points) Find a trig substitution $x = f(\theta)$ that makes

$$\int \frac{1}{(\sqrt{4+x^2})^3}\,\mathrm{d}x = \frac{1}{4}\int \cos\theta\,\mathrm{d}\theta.$$

Q2 - Week 7 (5 points)

(1) (1 point) Circle all of the following integrals that are **improper**. No justification is needed.

- $\displaystyle\int_4^7 \ln(x-3)\,\mathrm{d}x$
- $\displaystyle\int_1^7 \frac{1}{x-2}\,\mathrm{d}x$
- $\displaystyle\int_1^{-1} \frac{\sqrt{x+2}}{2-x^2}\,\mathrm{d}x$
- $\displaystyle\int_1^3 \frac{3}{x^2-3x}\,\mathrm{d}x$

(2) (2 points) Does $\displaystyle\int_1^\infty \frac{x}{e^{x^2}}\,\mathrm{d}x$ converge or diverge? Explain your answer.

(3) (2 points) Let a denote a real number. Find the value(s) of a such that $\displaystyle\int_1^\infty \frac{ax}{e^{x^2}}\,\mathrm{d}x$ converges to e. If no such a exists, explain why. You may use your work from part 2.

Q3 - Week 5 (5 points)

Let f be a function on $[0, \pi]$ whose second order derivative is continuous and $f(\pi) = 1$. Suppose that

$$\int_0^\pi (f(x) + f''(x)) \sin x \, \mathrm{d}x = 4.$$

Compute $f(0)$.

Hint: First break up the integral into two integrals.

Q4 (5 POINTS)

(1) (3 points) Using the method of partial fractions, compute the indefinite integral

$$\int \frac{x}{3x - 2x^2 - 1}\,\mathrm{d}x.$$

(2) (2 points) Evaluate

$$\int_{-1}^{1} \frac{xe^{x^2} + 1 + x^2}{(1 + x^2)^2}\,\mathrm{d}x.$$

Hint: Write this as two "easy" integrals. You do not need u-substitution or integration by-parts.

[End of Term Test 2]