THE UNIVERSITY OF TORONTO MISSISSAUGA
DECEMBER 2020 FINAL EXAMINATION
MAT135H5 F- Differential Calculus

Instructors: N. Askaripour, J. Herman, H. Horowitz, J. de Jong, H. Lamei Ramandi, M. Pawliuk, S. Shen, J. Yang, Z. Wang.
Date: December 19, 2020
Time: 9:00am - 12:00pm (EST)
Duration: 3 hours: (2 hours for writing, 1 hour for digitizing and uploading)
Aids: Course Textbook, Course Notes, non-programmable calculator

Submission

- **You must submit your completed solutions on Crowdmark by 12:00pm EST Saturday December 19, 2020**. Late submissions will not be accepted.
- You should start uploading your solutions no later than 11:00am.
- If you require additional space, please insert extra pages.
- You do not need to print out this test; you may submit clear pictures/scans of your work on lined paper, or screenshots/PDFs of your work.

Additional Instructions

- Between 8:00am-1:00pm on the day of the exam you must fill out the declaration of academic integrity at `http://declaration.utm.utoronto.ca/` and enter "MAT135HF" and your full legal name. **If you do not do this you may receive a penalty of 10% on your final exam**.
- You must justify and support your solution to each question.
- If you have any issues during the exam, please contact: `m.pawliuk@utoronto.ca` before the deadline.

Permitted Resources

During the test:

- You may use any resources (course notes, textbook, videos) that have been posted to Quercus by instructors or TAs.
- You may use a non-programmable calculator.
- You may use personal notes related to official course material (from reading the textbook, participating in lectures/tutorials, posted course videos, completing WeBWork and Written Assignments).
- You may contact the instructors on Piazza using a private post.
- Do not use personal notes related to other material (e.g. notes created by studying external websites)
- Do not communicate with anyone other than the instructors.
- Do not use any online resources other than Piazza, Quercus, and Crowdmark.

Academic Integrity

Between 8:00am-1:00pm on the day of the exam you must fill out the declaration of academic integrity.

- Go to `http://declaration.utm.utoronto.ca`
- Sign-in using your U of T login.
- Enter "MAT135HF" and your full legal name.

If you do not do this you may receive a penalty of 10% on your final exam.

You should not discuss this test with anyone else while the test is happening.
By submitting this test I affirm that this test represents entirely my own efforts. I confirm that:

- I have not copied any portion of this work.
- I have not allowed someone else in the course to copy this work.
- I did not discuss this test with anyone during the test.
- I understand the consequences of violating the University's academic integrity policies as outlined in the *Code of Behaviour on Academic Matters*.

Submission Checklist

This does not need to be submitted. It is for your own use.

☐ I have completed the declaration of academic integrity at `http://declaration.utm.utoronto.ca`

	Finished	Digitized	Uploaded in correct place, order, and orientation
Question 1	☐	☐	☐
Question 2	☐	☐	☐
Question 3	☐	☐	☐
Question 4	☐	☐	☐
Question 5	☐	☐	☐
Question 6	☐	☐	☐
Question 7	☐	☐	☐
Question 8	☐	☐	☐
Question 9	☐	☐	☐
Question 10	☐	☐	☐
Question 11	☐	☐	☐
Question 12	☐	☐	☐

☐ I have submitted everything in the right place, by the deadline. Time to rest.

Long Answer

The following 12 questions are worth five (5) marks each for a total of 60 marks.
Show your work. Unsupported solutions will receive little or no credit.

Question 1. [5 MARKS]

Consider the following five functions.

- $a(x) = x - 2$
- $b(x) = |x|$
- $c(x) = x + 2$
- $d(x) = -x$
- $e(x) = x + 4$

Let $f(x) = e \circ d \circ c \circ b \circ a(x)$. (Note: $f(x)$ is the composition of the five functions a, b, c, d, e.)

1. Write the function $f(x)$ as a piecewise function with two parts.

2. Sketch the graph of $f(x)$.

Question 2. [5 MARKS]

Let N be your University of Toronto student number. For convenience of the graders, please write your student number here:

$N =$ _____

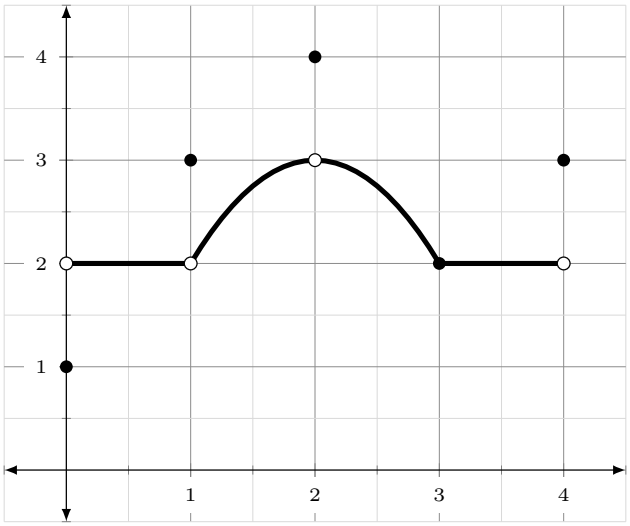
Find the domain of the function

$$f(x) = \frac{\sqrt{N - \ln(x)}}{x - e}$$

.

Question 3. [5 MARKS]

Let $f(x)$ be described by the graph below.



Compute the following limits, if they exist. (For Parts 1 and 2, no justification is needed. For Part 3, justify your answer.)

1. $\lim_{x \to 2} f(x)$
2. $\lim_{x \to 0^+} f(x)$
3. $\lim_{x \to 3} f(f(f(x)))$

Question 4. [5 MARKS]

Find all horizontal asymptotes of the function

$$f(x) = \frac{e^{2x} + \frac{1}{x} + 2}{3 - e^x}.$$

You should justify that your answer(s) are horizontal asymptotes, and that you have found all horizontal asymptotes.

Question 5. [5 MARKS]

Find the derivative of $f(x) = \sqrt{x^2 + 1}$ at $a = 2$, using the limit definition (i.e. first principles).

Question 6. [5 MARKS]

Let N be your University of Toronto student number and $f(x) = e^{x-1}(b - x)$.

For convenience of the graders, please write your student number here: $N =$ _____

1. Find $f'(x)$, $f^{(2)}(x)$, and $f^{(3)}(x)$.

2. Find b such that
$$f^{(N+2020)}(2N) = 0.$$

Question 7. [5 MARKS]

Find the tangent line at $(1, 0)$ of the curve

$$xe^y = 2\sin(xy) + \cos y.$$

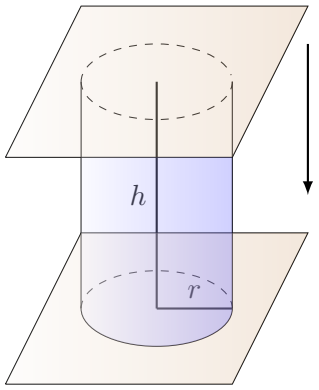
Question 8. [5 MARKS]

You are creating a dessert called a "S'more" as follows:

- You start with a (cylindrical) marshmallow with height 3 cm and radius 1 cm.
- You put a marshmallow between two flat cookies, so that the flat parts of the marshmallow are against the cookies. (See picture.)
- You press down on the top cookie at a constant rate of 0.5 cm/s, which flattens and compresses the marshmallow.
- You may assume that the marshmallow stays a cylinder through the compression, and its volume does not change.

Find the radius at the time when the rate the radius of the marshmallow is growing is the same as the rate the height of the marshmallow is shrinking.

The volume V of a cylinder of radius r and height h is $V = \pi h r^2$.



Question 9. [5 MARKS]

Compute the limit:

$$\lim_{x\to 0}(1+3x)^{\frac{-5}{x}}.$$

Question 10. [5 MARKS]

Consider the function $f(x) = 2x^{\frac{3}{2}} + 36\sqrt{x} - 15x - 7$ (where $x \geq 0$).

1. Find where f is decreasing and where it is increasing. (Note: give all your intervals as open intervals.)

2. Find all local maximums and local minimums of $f(x)$. Give only the x values. (Note: Do not consider the endpoints.)

Question 11. [5 MARKS]

Alice plans to sell cylindrical cans of lemonade. She can afford to use 220cm^2 of aluminum per can. What should the radius and height of the cans be in order to maximise the volume of lemonade per can?

You may find the following expressions for the volume, V, and surface area, SA, of a cylinder helpful:

$$V = \pi r^2 h,$$
$$SA = 2\pi(r^2 + rh)$$

where r is the radius of the base, and h is the height.

Question 12. [5 MARKS]

Suppose that $F(x)$ is an antiderivative of $f(x) = \dfrac{(\sec^2 x)e^x - (\tan x)e^x}{(e^x)^2}$.

1. What is $F'(x)$?

2. Find $F(x)$. You may give any antiderivative of $f(x)$. (Hint: Does $f(x)$ remind you of the product rule, the power rule, the chain rule, or the quotient rule?)

Total Marks = 60