MAT135H5 F - FALL 2020 - WRITTEN ASSIGNMENT 2

Submission

- **You must submit your completed Written Assignment on Crowdmark by 6:00pm (EDT) Friday October 30, 2020**. You will be emailed a link from Crowdmark with information on how to submit your solutions.
- Late assignments (even by a couple seconds) will not be accepted.
- Consider submitting your assignment well before the deadline.
- You do not need to print out this assignment; you may submit clear pictures/scans of your work on lined paper, or screenshots of your work.
- You do not need to submit the cover page, or the grading scheme.
- You must correctly orient/rotate and order your submission.
- If you require additional space, please insert extra pages.

Additional Instructions

You must justify and support your solution to each question. You should use full sentences.

Academic Integrity

You are encouraged to work with your fellow students while working on questions from the written assignments. However, the writing of your assignment must be done without any assistance whatsoever. Do not post partial or complete solutions to Piazza.

I affirm that this assignment represents entirely my own efforts. I confirm that:

- I have not copied any portion of this work.
- I have not allowed someone else in the course to copy this work.
- This is the final version of my assignment and not a draft.
- I understand the consequences of violating the University's academic integrity policies as outlined in the *Code of Behaviour on Academic Matters.*

By submitting solutions for grading I agree that the statements above are true. If I do not agree with the statements above, I will not submit my assignment and will consult the course coordinator (Mike Pawliuk) immediately.

Grading Scheme

This is the grading scheme that TAs will use when grading this assignment. You do not need to submit this page.

Question 1. [5 points]. Part 1 is worth 2 points: 1 point for a correct mathematical setting to solve the problem, 1 point for a correct answer given in a full sentence. Part 2 is worth 3 points: 1.5 points for a correct mathematical setting to solve the problem, 1 point for correct computations, and 0.5 point for a correct answer.

Question 2 [4 points]. Part 1 and Part 2 are worth 1 point for each for correct answers given in full sentences. Part 3 is worth 2 points: keep in mind that there is no unique answer for this part. Write your own thoughts that are supported by the Intermediate Value Theorem. As long as your writing is clear and complete you will get 1 point regardless of the contents. Another 1 point is for a correct use of the Intermediate Value Theorem.

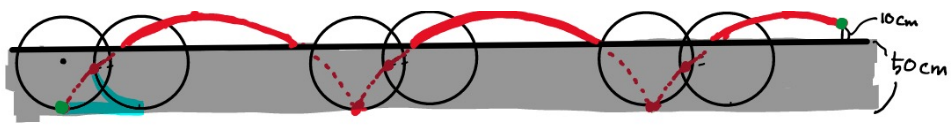
Question 3 [5 points]. This question has four parts. The first three parts have 1 point each. For the fourth part, which has 2 points, writing the definition of the derivative of the function at $x = 0$ has one point. Calculating that limit has 1 point.

Question 4 [3 points]. This question has two parts. The first part has 2 points. Writing the area of the triangle as a function depending on t has 1 point. Another point is for finding the rate of the change. Part 2 of this question has 1 point.

Question 1. (5 points)

You are a detective who needs to solve a murder case. The following is the list of clues that you have collected so far.

- The suspect ran away from the murder scene by bike.
- The suspect did not know that there was a button stuck to one of the wheels, which left a unique mark on the ground at the scene. The same mark was found on a straight road 70π centimeters from the scene (yes, you are extremely careful in measuring.), which was made the first time the button touched the ground again.
- A witness saw the button moving as the suspect fled the scene from the bottom of his window, which is 50 centimeters above the ground. When the button appeared for the third time its arc moved down to a point 10 centimeters above the bottom of his window before disappearing for good. See the figure below.



Hint: Use a cycloid (`https://en.wikipedia.org/wiki/Cycloid#:~:text=In%20geometry%2C%20a%20cycloid%20is,curve%20rolling%20on%20another%20curve.`)

(1) (2 points) Find the diameter of the wheel.

(2) (3 points) How far was the suspect from the scene when the witness saw the button for the last time?

Question 2. (4 points) Intensive harvesting of a population of a fish species can cause population extinction. We consider the following two models for harvesting of a given fish species and analyze how the extinction depends on the nature of the harvesting. The population size P (measured in thousands) is a function of harvesting effort h. (The harvesting effort is a mathematical measure of "fishing effort", which you are not expected to know in details.)

$$\text{Model 1:} \qquad P(h) = \begin{cases} 3(1-h) & \text{if } 0 \le h \le 1 \\ 0 & \text{if } h > 1 \end{cases}$$

$$\text{Model 2:} \qquad P(h) = \begin{cases} 1+\sqrt{4-3h} & \text{if } 0 \le h \le \frac{4}{3} \\ 0 & \text{if } h > \frac{4}{3} \end{cases}$$

(1) (1 point) What is the initial population of this species when no harvesting efforts were applied at all?

(2) (1 point) Draw the graph of each model.

(3) (2 points) Here you will analyze each model in terms of the Intermediate Value Theorem by answering the following questions. Which model has a situation where a small change in harvesting effort causes a sudden extinction? In Model 2, is there a harvesting effort to obtain the population of 500?

Question 3. Assume $L_a(x)$ is the line that is tangent to the graph of $y = f(x) = x^2$ at $(a, f(a))$. Let A_a be the point at which $L_a(x)$ intersects the line $x = 0$.

(1) (1 point) Find the equation for $L_a(x)$ for each $a \in \mathbb{R}$.

(2) (1 point) Compute the distance between the points A_x and $(x, f(x))$.

(3) (1 point) Find the rate of the change of the distance between A_x and $(x, f(x))$ with respect to x.

(4) (2 points) Use the definition of derivative to show that this rate is not defined at $x = 0$.

Question 4. Alice and Bob start at the origin. They start to run and their positions at time $t \geq 0$ are described as follows:

- Alice runs along the x axis and she is located at $(f(t), 0)$ at time t,
- Bob runs along the y axis and he is located at $(0, g(t))$ at time t.

(1) (2 points) If $f(5) = 10, f'(5) = 2, g(5) = -2 = g'(5)$ find the rate of the change of the area of the triangle OAB (formed by the origin, Alice, and Bob) at $t = 5$.

(2) (1 point) Show that if $f(t_0) = f'(t_0) = 0$, then the rate of the change of the area of the triangle is 0 at $t = t_0$.