MAT135H5 F - FALL 2020 - WRITTEN ASSIGNMENT 3 - UPDATED NOVEMBER 2

Updates and corrections

November 2:

- Q2.3 should say "$-n\cos^{n-1}(x)\sin((n+1)x)$".
- Q4.1.2 should say "then differentiate and isolate for y'.", not y.

Submission

- **You must submit your completed Written Assignment on Crowdmark by 6:00pm (EST) Friday November 13, 2020**. You will be emailed a link from Crowdmark with information on how to submit your solutions.
- Late assignments (even by a couple seconds) will not be accepted.
- Consider submitting your assignment well before the deadline.
- You do not need to print out this assignment; you may submit clear pictures/scans of your work on lined paper, or screenshots of your work.
- You do not need to submit the cover page, or the grading scheme.
- You must correctly orient/rotate and order your submission.
- If you require additional space, please insert extra pages.

Additional Instructions

You must justify and support your solution to each question. You should use full sentences.

Academic Integrity

You are encouraged to work with your fellow students while working on questions from the written assignments. However, the writing of your assignment must be done without any assistance whatsoever. Do not post partial or complete solutions to Piazza.

I affirm that this assignment represents entirely my own efforts. I confirm that:

- I have not copied any portion of this work.
- I have not allowed someone else in the course to copy this work.
- This is the final version of my assignment and not a draft.
- I understand the consequences of violating the University's academic integrity policies as outlined in the *Code of Behaviour on Academic Matters*.

By submitting solutions for grading I agree that the statements above are true. If I do not agree with the statements above, I will not submit my assignment and will consult the course coordinator (Mike Pawliuk) immediately.

Grading Scheme

This is the grading scheme that TAs will use when grading this assignment. You do not need to submit this page.

Question 1. [5 points].

- Part 1 is worth 2 points. 0.5 points for the correct y-intercepts. 1 point for correct application of implicit differentiation. 0.5 points for correct computation of the final answer.
- Part 2 is worth 2 points. 1 point for correctly setting up the equations. 1 point for finding the correct x and y coordinates.
- Part 3 is worth 1 point. Only the final answer is graded.
- 1 point total may be removed if the solution does not use complete sentences.

Question 2 [5 points].

- Part 1: 1.5 points (0.5 for dealing with the absolute value appropriately, 0.5 for arriving at the answer with correct and reasonable steps (that is, not skipping a crucial non-trivial equality), 0.5 for explaining their reasoning adequately).
- Part 2: 2 points (0.5 for using the product rule correctly, 0.5 for using the chain rule correctly, 0.5 for using the relevant trigonometric identity correctly), 0.5 points for concluding the special case.
- Part 3: 1.5 points (as in Part 2).
- 1 point total may be removed if the solution does not use complete sentences.

Question 3 [5 points].

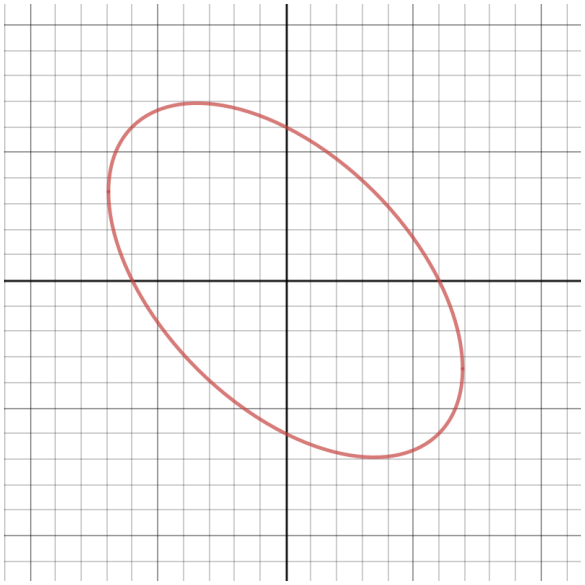
- Part 1 is worth 1 point. 0.5 points for the correct set up. 0.5 points for the final answer.
- Part 2 is worth 1 point.
- Part 3 is worth 3 points. 1 point for figuring out when their shadows have the same length. 1 point for correctly applying implicit differentiation. 1 point for computing the final answer.
- 1 point total may be removed if the solution does not use complete sentences.

Question 4 [5 points].

- Parts 1.a, 1.b, and 2 are worth 1 point each for a complete and correct solution using complete sentences.
- Part 3 is worth:
  - 1 point for a reasonable and plausible strategy that is explained in an introduction sentence, and
  - 1 point for a complete and correct solution using full sentences that contains a conclusion sentence.

Question 1. Consider the slanted ellipse with equation $x^2 + y^2 + xy = a^2, a \neq 0$.

(1) Find all y-intercepts. Compute the tangent line at those y-intercepts.
(2) Find all points on the graph with horizontal tangent line.
(3) Find all points on the graph with vertical tangent line.



Question 2. In all parts of this question, n is a natural number.

(1) Show that $\frac{d}{dx}|\sin(x)| = \frac{\sin(x)\cos(x)}{|\sin(x)|}$ whenever $\sin(x) \neq 0$.

(2) Show that $\frac{d}{dx}(\sin^n(x)\cos(nx)) = n\sin^{n-1}(x)\cos((n+1)x)$.
Conclude that $\frac{d}{dx}(\sin^{2020}(x)\cos(2020x)) = 2020\sin^{2019}(x)\cos((2021)x)$.

(3) Show that $\frac{d}{dx}(\cos^n(x)\cos(nx)) = -n\cos^{n-1}(x)\sin((n+1)x)$.

Question 3. Bob who is 6 feet tall is walking away from a lamp post to the east at a speed of 6 feet per second. When bob is 3 feet from the lamp post, his shadow is 2 feet long.

(1) Compute the height of the lamp post.

(2) How fast is the length of his shadow increasing?

(3) Alice who is 5 feet tall stands right below the lamp post. When Bob is 6 feet from the lamp post, she starts running to the south at a speed of 10 feet per second. How fast is the distance between Alice and Bob increasing when their shadows have the same length?

Question 4.

(1) Compute the derivative of $y = (x+1)(x+2)$ in two ways:
   (a) First expand, and then use the power rule.
   (b) Take the natural log of both sides, then differentiate and isolate for y'.
(2) By algebraic manipulation, show that the two versions of y' you computed in part (1) are equal.
(3) Compute the derivative of $y = (x+1)(x+2)(x+3)\cdots(x+2020)$ in any way possible. (In your solution include a 1-2 sentence introduction and conclusion similar to Tutorial activity 2.)